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Engineering Notes

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Application of Sensitivity Analysis for Optimization of a Satellite Structure

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Nomenclature

F = implicit function of design variables
g = acceleration of gravity
R = set of real numbers
X = set of design variables
δ = variation
λ = natural frequency of the structure, Hz
∇_X = gradient operator with respect to design variables

Introduction

DESIGN sensitivity analysis has been used in the solution of small academic problems for over 30 years.[1] Application to complex structural systems was prevented because of the high costs of the finite difference schemes, the only numerical technique used to perform structural design sensitivity analysis in the past. Also, precision issues are major concerns in the application of the finite difference method.

Recently, very efficient direct differentiation and adjoint variable design sensitivity analysis formulations have been developed for a large class of structural problems.[2–4] Several of these formulations, which add only a fraction of the structural analysis cost over the total computational cost, are now implemented in commercial software, making optimization of complex structural systems feasible.

Here we apply sensitivity analysis in the optimum design of the Brazilian Scientific Satellite, SACI-1, whose structure is a $600 \times 400 \times 400$ mm parallelepiped, weighing about 65 kg. The structure is divided into four parts (Fig. 1a): the main body (a pack of nine ribbed aluminum frames), the sensor bay (a box of six ribbed plates where most of the payload is assembled), the four deployable solar panels of 440×570 mm each, and the adapter cone (an interface between the satellite and the launcher).[5]

A finite element model (FEM), based on a starting structural design and adjusted by sine-burst, sinusoidal, and random vibration tests on a prototype, was used for the structural analysis and redesign and to predict the vibration levels on the most delicate electronic components.

The finite element analysis results indicated that the accelerations on the electronic equipment were exceedingly high. The lowest longitudinal natural frequency of the structure was very close to the fundamental frequency of the frames where the printed circuit boards were assembled, but well above the minimum allowed by the launcher's specification. The lowest lateral natural frequencies were also much larger than the minimum allowable. The total mass of the structure, on the other hand, was at the maximum allowable.

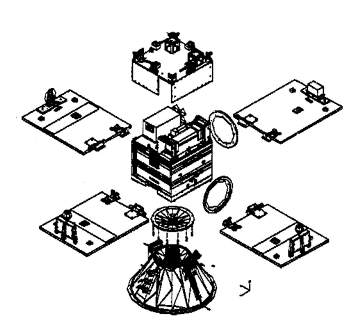
Thus, the following problem was posed: change the original design to reduce the lowest longitudinal natural frequency of the structure, while reducing its mass.

In this Note we show how design sensitivity analysis can be used to solve the problem in an efficient manner. First, we describe the original numerical model and the finite element analysis. Then, we define the optimization problem and show, in a systematic way, how design sensitivity analysis was used to make the design changes in the optimization process more effective.

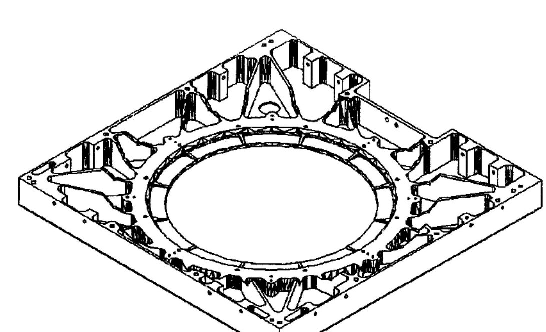
Finite Element Analysis

The FEM, with 5251 nodes and 5838 elements, was analyzed using MSC/NASTRAN. A modal superposition technique with a cutoff frequency of 600 Hz was used to reduce the cost of the dynamic analysis for which all instruments were modeled as concentrated masses and the components of the electronic boards, as distributed nonstructural masses.

The frequencies of the first two lateral (modes 1 and 2) and the first longitudinal (mode 5) vibration modes are presented in Table 1,



a) Exploded view of the structure



b) Mechanical interface modulus

Fig. 1 Satellite's structure.

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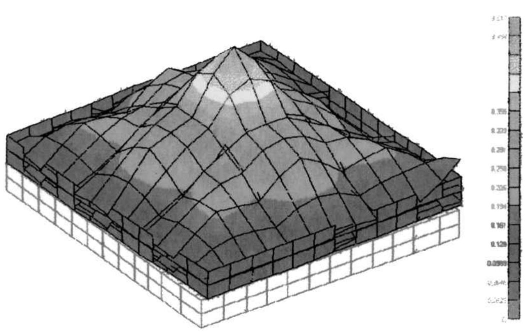
Table 1 Numerical and test results of the natural frequencies

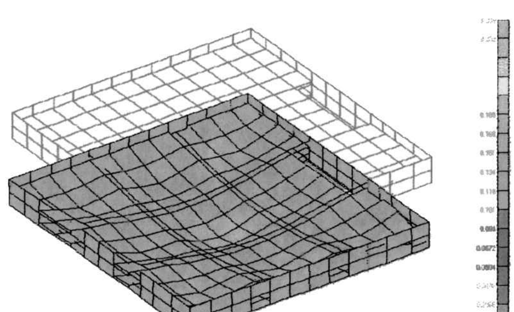
	Result, Hz	
Vibration mode	MSC/NASTRAN	Test
1	51	51
2	52	51
5	162	165

Table 2 Natural frequencies of a typical frame from low-level sine sweep test

Number	Frequency, Hz	Acceleration ($\times g$)
1	181.87	1.445
2	394.72	11.529
3	1038.05	4.439
4	1283.09	9.751



a) Nonoptimized frame



b) Optimized frame

Fig. 2 Vibration mode of Orcas ribbed frame.

in comparison with those obtained in the lab tests.[6] The fifth mode revealed an amplification of displacements at the central region of all of the ribbed aluminum frames where the printed circuit boards were assembled (Figs. 2a).

A typical aluminum frame was constructed and tested at the laboratory to determine the first four natural frequencies in the z axis.[7] The results shown in Table 2 indicated that the natural frequency of 162 Hz associated with the fifth mode was too close to the first natural frequency of 182 Hz (in the z direction) of the ribbed frames. Therefore, unwanted amplifications are likely to occur in the ribbed frames during launching, which might seriously damage the electronic circuitry of the printed circuit boards assembled on them.

Next, we describe the optimization problem and the use of sensitivity analysis in the process.

Design Optimization

Description of the Problem

The problem consists of resizing the structural members in order to reduce the lowest longitudinal natural frequency to minimize the amplifications described in the preceding section. The material and section properties of all of the 5838 elements can be considered in the design variables set. Fortunately, lumping of material and section properties reduces drastically the design space. Because we do not consider changes of material properties, the design space is further reduced to a total of 173 design variables, divided into three groups: 120 beam element sectional dimensions, 35 shell element thicknesses, and 18 design variables controlling the shapes of the mechanical interface ribs (Fig. 1b).

Now, we ask the following questions:

1) Which design variables should be resized so that the fifth natural frequency is reduced?

2) How much should we change the size of a given design variable?

3) Should we increase or decrease the size of a given design variable?

4) How much will those changes affect the other natural frequencies?

5) What restrictions on the response of the structure would be violated by the design changes?

6) Will the fifth mode of the modified structure still be a longitudinal vibration mode?

For a complex structure like this, it is very difficult and time consuming to answer all of these questions based on trial and error, even for an experienced analyst.

Reduction of the Design Space

Design Sensitivity Analysis

Design sensitivity analysis helps us to answer some of those questions, making it possible to further reduce the design space, as follows. First, we denote λ_5 the fifth natural frequency of the initial design and $\boldsymbol{X}$ the set of number of design variables (NDV):

$$X = \{x_i \in R \mid 1 \leq i \leq \text{NDV}\} \tag{1}$$

Then we call $F : \boldsymbol{X} \rightarrow R$ the implicit mapping of a point in design space to the fifth natural frequency

$$\lambda_5 = F(\boldsymbol{X}) \tag{2}$$

Hence, each term of the design sensitivity array $\nabla_x F(\boldsymbol{X})$ in the expression

$$\delta\lambda_5 = \nabla_X F(\boldsymbol{X}) \cdot \delta\boldsymbol{X} \tag{3}$$

represents the change in λ_5 caused by a small variation on the corresponding design variable.

Typical sensitivity analysis results are shown in Tables 3 and 4. Based on such results, we eliminated the following from the design space: 1) each design variable associated with a negative sensitivity term because these indicate that a reduction on λ_5 can be achieved only by increasing the mass of the structure; and 2) each design variable whose sensitivity term is less than the threshold of $1.E+05$

Table 3 Sensitivities of λ_5 with respect to the ribs shape design variables

Number	Name	Initial value, mm	Sensitivity
1	Z1197	29.0	$4.31E+06$
9	Z1185	16.1	$-2.25E+05$

Table 4 Sensitivities of λ_5 with respect to the plate thickness design variables

Number	Name	Initial value, mm	Sensitivity
3	PS1005	1.50	$1.00E+07$
*14	PS2011	3.00	$1.33E+07$
20	PS4003	2.00	$1.70E+04$
21	PS5002	2.00	$-9.66E+04$

(these design variables are shown on the tables with an underlined font). The set of design variables dropped from 173 to 89 design variables (14 shape variables, 22 plate thickness variables, and 53 beam sectional dimensions).

Design Restrictions

Among the 89 remaining design variables, PS2011, PS7002, and PS9000, (thicknesses of external plates) are eliminated from the design space because they shield the interior components of the satellite and cannot be less than 3 mm (these variables are marked with an asterisk in Table 4). All of the design variables related to the frames where the electronic components were assembled were also eliminated. The idea was to leave the natural frequency of the frames unchanged to allow for uncoupling with the lowest longitudinal frequency of the structure. After this, the final design space was reduced to only 33 design variables.

Optimization with Reduced Space

After the reduction of the design space from a total of 173 to only 33 design variables, we could perform a much cheaper design optimization.

It is important to notice that, as the design changes with the optimization steps, the natural frequencies and corresponding vibration modes of the new design may not correspond to the vibration modes of the previous design. Thus, our original objective function λ_5 (the frequency of the first longitudinal vibration mode) may have to change, say to λ_3, a few optimization steps later. To avoid having to stop the optimization process to check for any changes in vibration mode, we changed the objective function to be the sum of the third, fourth, and fifth natural frequencies. Therefore, the final optimization problem is stated as follows:

Minimize

$$(\lambda_3 + \lambda_4 + \lambda_5)$$

Subject to

$$\lambda_1, \lambda_2 \geq 15{,}791(20\text{ Hz}), \qquad \lambda_3, \lambda_4, \lambda_5 \geq 98{,}696\,(50\text{ Hz})$$

$$\boldsymbol{X}_{\min} \leq \boldsymbol{X} \leq \boldsymbol{X}_{\max} \tag{4}$$

The results of the optimization problem of Eq. (4) obtained by solution 200 of MSC/NASTRAN with the method of feasible directions are shown in Fig. 2b and in Tables 5–7. The first natural longitudinal frequency was reduced from 162 Hz of the original design to the minimum of 88 Hz in five optimization steps. In the optimum design the first two natural vibration modes were still lateral vibration modes, but the first natural longitudinal vibration mode became the third vibration mode.

Table 5 Comparison of natural frequencies of the original vs the optimized structure

	Frequency, Hz	
Vibration mode	Original structure	Optimized structure
Lateral x direction	51	31
Lateral y direction	52	35
Longitudinal	162	88

Table 6 Initial and final values of shape design variables

	Value	
Design variable	Initial, cm	Final, mm
Z1197, Z1145	2.90	2.00
Z1658, Z1659	3.50	2.00
Z1182, Z1169	3.30	2.00
Z1148, Z1151, Z1166, Z1167, Z1174, Z1179, Z1206, Z1209	1.61	2.00

Table 7 Initial and final values of the plate thickness design variables

	Value, mm	
Design variable	Initial	Final
PS1001	2.00	3.00
PS1005	1.50	2.99
PS1006	1.30	3.00
PS1010	1.90	3.00
PS1011	5.20	5.20
PS1002,1012–1017, 9005, 9010, 9012	3.00	3.00
PS2012, 2019–2021	2.00	2.00

Conclusions

The efficient design sensitivity analysis formulations implemented in commercial structural analysis software are extremely helpful in the optimization process of complex structures. Therefore, regardless of the optimization strategy adopted, use of design sensitivity information prior to the optimization process is recommended.

In this Note we used the design sensitivities of the fifth eigenvalue in order to select a subset of the design variables to use in the optimization process. With design sensitivity information only, we were able to reduce the design space from 173 to a subset of only 89 design variables. First, we eliminated all of the design variables whose sensitivity terms had negative signs to avoid increasing the mass of the structure. Then, we eliminated all of the design variables below a selected threshold value (in our case 10^{+5}). Further reduction of the design space was achieved by examining the remaining set of design variables to exclude those for which some design restrictions would apply. The final design space was then reduced to only 33 design variables.

With the reduced set of design variables, the optimization problem was solved. The lowest longitudinal frequency was minimized from the original value of 162 Hz to the optimized value of 88 Hz. Without violating the lower boundary requirements of the frequencies, the first two global natural frequencies were also decreased from 51 and 52 Hz to 31 and 35 Hz, respectively.

Acknowledgments

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